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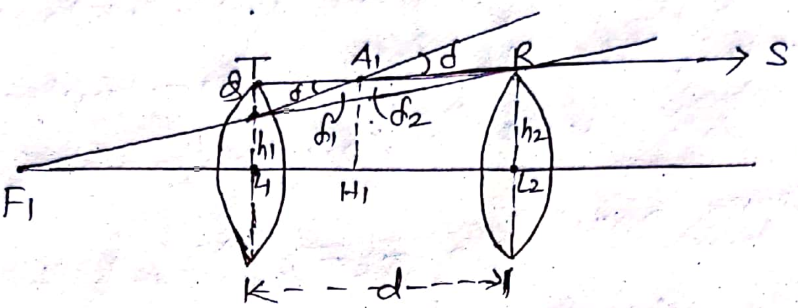
B.Sc (Part-II) Hons

Expression for principal points and focal point of a System of two lenses placed co-axially 'd' distance apart.

Position of the first principal point:—



Let us an incident ray F_1Q passing through the first focal point F_1 of the Combination. It follows the path QR and RS, becoming parallel to the axis. The incident ray produced forward and the final emergent ray produced backward meeting in a point A_1. The plane through A_1 and perpendicular to the axis is the first principal plane and its point of intersection with the axis is the first principal point. Hence H_1 is the first principal point. Hence H_1F_1 is the first equivalent focal length.

Let δ_1 and δ_2 be the deviations suffered at L_1 and L_2 respectively and δ the total deviation. Let h_1 and h_2 be the height at which the rays F_1R and QR meet the lenses L_1 and L_2 respectively.

$\therefore \delta_1 = \frac{h_1}{f_1}$ and $\delta_2 = \frac{h_2}{f_2}$

In order to produced the same deviation δ in a ray which emerges parallel to the axis, the equivalent lens should be placed in the position A_1H_1 and must have a focal length $(H_1F_1 = F)$

Then $\delta = \frac{h_2}{F}$

The distance H_1 from L_1 is given by

$$L_1H_1 = TA_1 = \frac{QT}{\tan\delta}$$

$$= \frac{RT\tan\delta_2}{\tan\delta}$$

$$= \frac{d\,\delta_2}{\delta} \quad (\because \delta \text{ and } \delta_2 \text{ are very small})$$

$$= \frac{d\,h_2/f_2}{h_2/F}$$

$$TA_1 = +\frac{dF}{f_2} \quad \text{--- (1)}$$

Thus the first principal point at a distance $(+Fd/f_2)$ from the first lens.

position of the First focal point:-

The distance of the first focal point F_1 from first lens L_1 is

$$L_1F_1 = H_1F_1 - H_1L_1$$

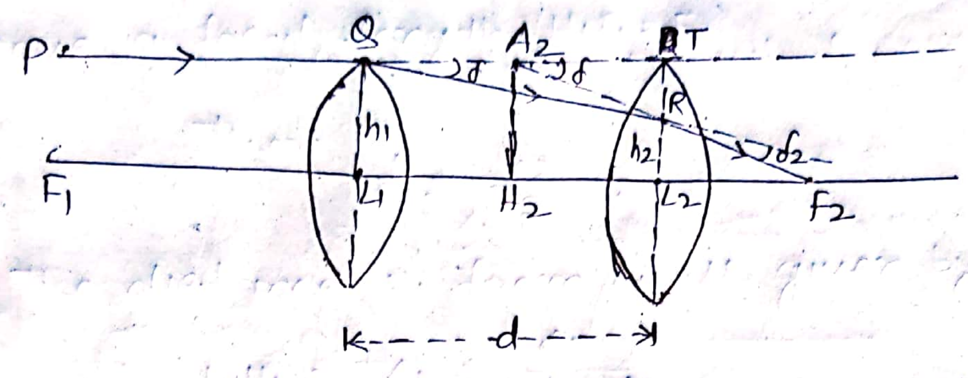
But $H_1F_1 = -F$ and $H_1L_1 = -L_1H_1$

$$= -Fd/f_2$$

$$\therefore \quad L_1F_1 = -F + \frac{Fd}{f_2}$$

$$L_1F_1 = -F\left(1 - \frac{d}{f_2}\right) \quad \text{——} \quad (2)$$

Position of the Second principal point:-



Let a ray PQ parallel to the principal axis be incident on the first lens L_1 at a height h_1 above the axis. On refraction through L_1 it suffers a deviation δ_1 and travells along QR, where

$$\delta_1 = \frac{h_1}{f_1}$$

The ray QR meets the lens L_2 at a height h_2 above the axis suffers a further deviation δ_2 in the same direction, where

$$\delta_2 = \frac{h_2}{f_2}$$

The final emergent ray meets the axis at F_2 which is the second focal point of the combination. Let the incident ray PQ produced, meet the second lens at T. Since both deviations are in the same direction the the total deviation δ is

$$\delta = \delta_1 + \delta_2$$

$$\delta = \frac{h_1}{f_1} + \frac{h_2}{f_2}$$

The incident ray PQ produced forward and the emergent ray RF_2 produced backward meet in a point A_2. Hence the plane A_2H_2 through A_2 is the Second principal plane and its point of intersection with the axis at H_2 is the Second principal point. Its distance from the Second lens L_2 is given by

$$L_2H_2 = TA_2 = -\frac{RT}{\tan\delta}$$

$$= -\frac{RT \tan\delta_1}{\tan\delta}$$

$$= \frac{-d \tan\delta_1}{\tan\delta}$$

$$= \frac{-d\,\delta_1}{\delta}$$

$$= \frac{-d\, h_1/f_1}{h_1/F}$$

$$TA_2 = -dF/f_1 \quad \text{--- (3)}$$

Thus the Second principal point is at a distance $(-Fd/f_1)$ from the Second lens.

Position of Second focal point:- The distance of the Second focal point F_2 from the Second lens L_2 is $L_2F_2 = H_2F_2 - H_2L_2$

But $H_2F_2 = +F$ and $H_2L_2 = -L_2H_2$

or, $H_2L_2 = (-Fd/f_1) = Fd/f_1$

$$\therefore L_2F_2 = F - \frac{Fd}{f_1} = F\left(1 - \frac{d}{f_1}\right) \quad \text{--- (4)}$$